Article

TRR
JOURNAL OF THE TRANSPORTATION RESEARCH BOARD

Transportation Research Record
2018, Vol. 2672(29) 1–11

DOI: 10.1177/0361198118780876
journals.sagepub.com/home/trr
SAGE

Data-Driven Modeling of Aircraft Engine Fuel Burn in Climb Out and Approach

Yashovardhan S. Chati[1] and Hamsa Balakrishnan[1]

Abstract

Fuel burn is a key driver of aircraft performance, and contributes to airline costs and emissions. Low-altitude fuel burn and emissions, such as those that occur during climb out and approach, have a significant impact on the environment in the vicinity of airports. This paper proposes a new methodology to statistically model fuel burn in the climb out and approach phases using the trajectory of an aircraft. The model features are chosen by leveraging a physical understanding of aircraft and engine dynamics. Model development is conducted through the use of Gaussian Process Regression on a limited Flight Data Recorder archive, which also provides ground truth estimates of the fuel flow rate and total fuel burn. The result is a class of models that provide predictive distributions of the fuel burn corresponding to a given aircraft trajectory, thereby also quantifying the uncertainty in the predictions. The performance of the proposed models is compared with other frequently used Aircraft Performance Models. The statistical models are found to reduce the error in the estimated total fuel burn by more than 73% in climb out and by 59% in approach.

The amount of fuel consumed, as well as the rate of consumption, are key aspects of an aircraft's performance. Fuel burn also constitutes a major component of the Direct Operating Costs of an airline, and results in emissions that have environmental and health impacts, especially in the vicinity of airports.

It is estimated that about 10% of all aircraft emissions, and 30% of aircraft carbon monoxide emissions, occur close to the ground (*1*). Climb out can contribute to as much as 6% of the total fuel consumed in a flight, while approach can contribute to as much as 4% of the total fuel consumed. It is therefore important to estimate the fuel burn (and subsequently, the emissions) in the climb out and approach phases of flight. While fuel flow profiles are typically monitored by sensors on board aircraft, they are not disseminated to entities outside the airline, including environmental inventories. By contrast, aircraft trajectory information (i.e., its physical location as a function of time) is available from surveillance systems. Motivated by these observations, this paper investigates the following problem: *Given the trajectory of a flight, can one infer its fuel burn profiles in the climb out and approach phases?*

The ability to translate aircraft trajectory information into fuel burn estimates forms an important component of fuel burn and emissions inventories. Such tools can be used by aviation researchers to assess the impact that the introduction of new takeoff and landing procedures will have on the environment around airports. Projection studies make use of aircraft performance models to forecast the emissions impacts of future demand growth (*2*).

Engine performance simulators (e.g., GasTurb (*3*)) typically use physics-based approaches to model fuel burn. Such approaches are helpful in design studies, since it is possible to control the underlying simulation variables needed as inputs to the model. However, they require a knowledge of engine operating points, and parameters such as the temperature and pressure ratios at any time, which are not readily available for operational flights. Previous efforts to develop data-driven models of fuel burn have typically used simulated or nonoperational data from flight manuals, ground tests, simulation, and performance calculators to model the fuel flow rate (4, 5). Such methods do not always capture the performance of real-world aircraft operations. Prior work has also shown that the International Civil Aviation Organization (ICAO) Aircraft Engine Emissions Databank (ICAO Databank) (*6*) can overestimate operational fuel burn and emissions, and that the total fuel burn in the Landing and Takeoff (LTO) cycle can be overestimated by up to 40% (*7*). Finally, there

[1]Massachusetts Institute of Technology, Cambridge, MA

Corresponding Author:
Address correspondence to Yashovardhan S. Chati: yschati@mit.edu

has been recent work categorizing different international efforts on fuel burn modeling, and describing the associated challenges (*8*).

An aircraft in flight is subject to random disturbances, such as manufacturing tolerances, component deterioration, air turbulence, and so forth. As a result, the aircraft is better modeled as a statistical system than as a deterministic one (*9*). A statistical model also enables the quantification of uncertainty in the fuel burn. These uncertainty estimates reflect the effect of random disturbances and modeling assumptions, and help quantify the variability seen in real operations. The quantification of the uncertainty in nominal performance can help identify and rectify anomalous behavior, thereby saving significant maintenance costs (*10*).

Contributions of This Paper

The primary contribution of this paper is the development of a methodology for the data-driven statistical modeling of aircraft fuel burn in climb out and approach. The fuel burn in these phases of flight has a particularly significant environmental impact on airports and their surroundings. The modeled variables are the instantaneous fuel flow rate at any time, and the total mass of fuel consumed (in climb out or approach). Both these quantities are important from an emissions perspective: some emissions (e.g., nitrogen oxides, hydrocarbons) depend on the instantaneous fuel flow rate, whereas others (e.g., carbon dioxide, water vapor) depend on the total fuel burn.

The modeling approach adopted in this paper attempts to use data to identify the mapping between the input and output variables. To identify such a mapping, a training dataset is required that contains the input (trajectory) variables for a set of flights, as well as the corresponding realized fuel flow profiles. In other words, a supervised learning approach (*11*) is adopted. The model will then, given a new set of trajectory variables, predict the corresponding fuel flow profile. The resulting predictive distribution will also reflect the variability seen in the training data and model uncertainty.

The proposed models leverage a physical understanding of aircraft and engine performance to identify the important features for fuel burn estimation. Reasonable assumptions are made to restrict the feature space to trajectory variables. The models are based on a statistical technique known as Gaussian Process Regression (*12*). The fuel burn estimates from the developed models are compared with those given by widely-used alternatives such as the Base of Aircraft Data (BADA), the Senzig-Fleming-Iovinelli (SFI), and the Boeing Fuel Flow Method 2 – corrected ICAO Databank (ICAO-BFFM2) models. The statistical models proposed in this paper are found to reduce errors in the estimation of the mean fuel flow rate by more than 62% in climb out and 34% in approach, averaged over the different aircraft types. Furthermore, these improved fuel flow rate estimates reduce the error of the estimated total fuel burn by more than 73% in climb out, and 59% in approach (averaged over the different aircraft types). It should be noted that, in this paper, climb out and approach are taken to be the parts of ascent and descent, respectively, taking place below 3000' Above Field Elevation.

Dataset

A supervised learning approach requires a training dataset that contains the input trajectory variables, as well as the realized fuel flow rate profiles. A dataset obtained from the Flight Data Recorders (FDRs) of operational flights of a European airline is used. The FDR is usually an accurate data source, since it records parameters during flight. The sampling frequency of the FDR data is 1 Hz during the takeoff, initial ascent, late descent and landing phases, and as low as 1/300 Hz in portions of the cruise phase. Eight different aircraft/engine types are represented in the dataset (Table 1): six Airbus aircraft types (whose engine types are derivable from their series and mark designators), Boeing B767-300 (run by GE CF6-80C2B7F engines), and B777-300ER (run by GE90-115B1 engines).

Model Features

The features of a model refer to the variables (and their transformations) that are included. Instead of a purely data-driven feature identification procedure, this paper leverages a physical understanding of aircraft dynamics. Four principal forces act on an aircraft in flight, namely, the thrust generated by the engines which propels the aircraft forward, the lift generated (primarily by the wings) which keeps it in the air, the drag or the resistance offered by the air, and the weight of the aircraft. Considering the aircraft as a point mass and using a simplified model of aircraft dynamics, the aircraft net thrust from all the engines (F_n) is given as

$$F_n = qSC_{D_0} + \frac{C_{D_2}m^2g^2}{qS} - \frac{C_{D_2}m^2g^2\dot{h}^2}{qSV^2} + mg\frac{\dot{h}}{V} + m\frac{dV}{dt}. \quad (1)$$

Here, S is the reference wing area (a constant for a particular aircraft type), C_{D_0} and C_{D_2} are aircraft drag coefficients, m is the aircraft gross mass, g is the acceleration due to gravity, $\dot{h}$ is the vertical speed, V is the true airspeed, and t

is the time. q is the aircraft dynamic pressure and equals $\frac{1}{2}\rho_\infty V^2$, where ρ_∞ is the ambient air density.

The average fuel flow rate per engine ($\dot{m}_f$) can be related to the net thrust via the Thrust Specific Fuel Consumption (TSFC) as

$$\dot{m}_f = \frac{\text{TSFC} \times F_n}{N_{\text{eng}}}, \tag{2}$$

where N_{eng} is the number of engines. For a particular engine type, the corrected TSFC can be assumed to be a function of the aircraft Mach number, the corrected engine parameters (such as corrected net thrust for the engine), and the engine component efficiencies (*13, 14*), and is given by

$$\frac{\text{TSFC}}{\sqrt{\theta_\infty}} = f_{\text{TSFC}}(M_\infty, \frac{F_{n,pe}}{\delta_\infty}, \nu_{\text{eng}}). \tag{3}$$

Here, $M_\infty = \frac{V}{\sqrt{\gamma R T_\infty}}$ is the aircraft Mach number, δ_∞ is the ambient pressure (P_∞) divided by the International Standard Atmosphere (ISA) sea level static pressure (101,325 Pa), θ_∞ is the ambient temperature (T_∞) divided by the ISA sea level static temperature (288.15 K), $F_{n,\,pe}$ is the averaged net thrust per engine, and ν_{eng} represents the engine component efficiencies. γ is the adiabatic constant for air and R is the gas constant for air. Equations 1–3 reveal the following functional dependency for the averaged per engine fuel flow rate for a given aircraft/engine type (neglecting constants for the particular aircraft/engine type):

$$\begin{aligned} &\dot{m}_f|_{\text{aircraft/engine type}} \\ &= f_{\dot{m}_f}(qS, m, \frac{\dot{h}}{V}, V, \frac{dV}{dt}, \sqrt{T_\infty}, C_{D_0}, C_{D_2}, \nu_{\text{eng}}) \end{aligned} \tag{4}$$

The aim of this study is to develop aircraft type specific models which can map the aircraft trajectory to fuel flow rate profiles. Trajectory data typically comprise the aircraft latitude, longitude, altitude, ground speed, climb rate, and course as functions of time. These models can then be used to estimate the fuel flow rate for a flight, given its trajectory from any surveillance system. The following simplifying assumptions are made to restrict the feature set, to the extent possible, to features which are extractable from trajectory data:

- Since high-fidelity weather data may not be available, the ambient air density, pressure, and temperature are assumed to be a function of the altitude, according to the ISA assumptions. Consequently, only the density is retained in the model to represent ambient weather conditions.
- Aircraft performance depends on the true airspeed (V), which is the aircraft velocity with respect to the surrounding air. Due to the effect of wind, this speed differs from the ground speed, namely, the speed relative to the ground. The trajectory of an aircraft, as observed by a surveillance system, only provides information on the ground speed. The calculation of the true airspeed requires additional wind data of the same spatial and temporal resolution as the trajectory information which is often unavailable. As the value of the true airspeed cannot be derived from the trajectory with the same fidelity as the other variables, it is not included as an input feature. Instead, the ground speed (V_{GS}), which can be derived from trajectory data, is used as a feature. It is expected that the neglect of the effects of wind will be reflected in larger variability, and consequently, larger prediction errors as well as prediction intervals.
- The rate of change of the true airspeed $\left(\frac{dV}{dt}\right)$ is also not observed in surveillance data. The numerical derivative of the ground speed ($\frac{\Delta V_{\text{GS}}}{dt}$)is therefore used as a feature. The values of the numerical derivative are smoothed through a low pass filter before using them for analysis to remove noise arising from numerical differentiation.
- The BADA assumptions on the drag coefficients are assumed to hold. In climb out, the drag coefficients are considered to be aircraft-type specific constants. In approach, the drag coefficients are assumed to have discrete levels depending on the altitude of the aircraft with respect to the mean sea level elevation of the arrival airport (h_{ATD}), the aircraft speed, and the aircraft gross mass. This dependence results from the different configurations the aircraft is in as it comes in to land (due to deployment of flaps, slats, landing gear) which change the aircraft drag coefficients.
- The engine component efficiencies depend on factors like the engine operating point, the engine age, and the level of maintenance. Because trajectory data alone do not give information on the engine component efficiencies or factors influencing the efficiencies, ν_{eng} is not included as a predictor variable.

As a result of the above assumptions, the functional relation in Equation 4 can be simplified to the following relation for a given aircraft/engine type:

$$\dot{m}_f|_{\text{aircraft/engine type}} \approx \begin{cases} \dot{m}_{f_{\text{(co)}}}(q_{\text{GS}}S, m, \frac{\dot{h}}{V_{\text{GS}}}, V_{\text{GS}}, \frac{\Delta V_{\text{GS}}}{\Delta t}), & \text{in climb out, and} \\ \dot{m}_{f_{\text{(ap)}}}(q_{\text{GS}}S, m, \frac{\dot{h}}{V_{\text{GS}}}, V_{\text{GS}}, \frac{\Delta V_{\text{GS}}}{\Delta t}, h_{\text{ATD}}), & \text{in approach.} \end{cases} \tag{5}$$

Here, the subscripts (co) and (ap) refer to the climb out and approach phases, respectively. q_{GS} is the dynamic pressure based on the ground speed, given by $\frac{1}{2}\rho_\infty V_{\mathrm{GS}}^2$. The predictor/input variables are therefore the dynamic pressure multiplied by the reference wing area ($q_{GS}\mathcal{S}$), the aircraft mass (m), the ratio of the vertical speed to the ground speed ($\frac{\dot{h}}{V_{\mathrm{GS}}}$), the ground speed ($V_{\mathrm{GS}}$), and the rate of change of the ground speed ($\frac{\Delta V_{\mathrm{GS}}}{dt}$). In approach, the aircraft altitude above the arrival airport elevation (h_{ATD}) is also taken as an additional predictor variable. The predicted/output variable is the aircraft per engine fuel flow rate ($\dot{m}_f$), averaged over all the engines. All the variable values are taken in SI units. It is important to note that except for the aircraft mass, all the predictor variables can be extracted from trajectory surveillance data. The cumulative effect of random disturbances, unmodeled factors, and different assumptions will be reflected in the fuel flow rate prediction intervals.

Statistical Models

Since all the variables chosen for the model are continuous, metric variables, mapping the aircraft trajectory to its fuel flow rate profile using a subset of labeled operational data comprises a supervised learning regression problem (*11*). In regression, the output of the i^{th} observation (y_i) is assumed to equal a function of the input features ($\mathbf{x_i}$) plus some noise (ε_i),

$$y_i = f(\mathbf{x_i}) + \varepsilon_i. \tag{6}$$

The goal is to estimate the underlying regression function ($f(\mathbf{x})$) using a set of given inputs and their corresponding given outputs. Once the regression function is estimated, it can be used to estimate the unknown output corresponding to a new input.

For the fuel flow rate modeling problem being investigated in this paper, a regression method is desired that is both nonparametric and probabilistic. Nonparametric methods adapt the regression function to the system complexity represented by the data, without requiring a choice of the form of the regression function a priori. Thus, nonparametric methods are useful in problems with no prior knowledge of the exact functional form of the feature vectors (which may be nonlinear). Probabilistic algorithms assume the noise terms to follow an underlying distribution. They can thus, give the complete predictive distribution of the output at a new input vector. A nonparametric, probabilistic regression method found to be useful in this paper is Gaussian Process Regression (GPR). GPR has found application in diverse areas, ranging from biomedical applications and health care (*15*) to music (*16*).

Gaussian Process Regression (GPR)

In GPR, the regression function ($f(\mathbf{x})$) is assumed to be drawn from a Gaussian Process (GP) with zero mean function and covariance/kernel function $k(\mathbf{x}, \mathbf{x}')$.

$$f(\mathbf{x}) \sim \mathcal{GP}(0, k(\mathbf{x}, \mathbf{x}')) \tag{7}$$

A function is said to follow a Gaussian Process when any finite set of function values follows a joint Gaussian distribution (*12*). The noise (ε_i) is also assumed to follow a Gaussian distribution. The function $k(\mathbf{x}, \mathbf{x}')$ is called the kernel function, and represents the covariance between the regression function values $f(\mathbf{x})$ and $f(\mathbf{x}')$ at two inputs $\mathbf{x}$ and $\mathbf{x}'$. The flexibility in GPR arises from the rich variety of possible kernel functions. Two kernel functions which have been found to be particularly useful in this study are the Dot Product Squared Exponential (DPSE) and the Dot Product Exponential (DPE) kernels. Kernel functions depend on *hyperparameters* which govern behavior such as their characteristic length-scales. Model building using GPR involves estimating these hyperparameters and thereby, the form of the kernel function. Details on kernel functions and hyperparameters can be found in prior work (*17*). Once the hyperparameters are estimated, GPR can be used to give the predictive distribution of the output at a new input vector. The assumptions of Gaussianity (Gaussian Process and Gaussian noise) lead to a Gaussian predictive distribution, resulting in mathematical tractability. More details of GPR can be found in (*12*).

Model Building

The FDR dataset for each aircraft type (for the climb out and approach phases) is divided into three parts, with flights randomly selected to belong to each subset. The training set (65% of flights) is used to build the model, the validation set (15% of flights) is used to select the 'best' model from a set of candidate models, and the test set (20% of flights) is used to evaluate the predictive performance of the model that is finally chosen.

Equation 5 shows that the aircraft mass is an input/predictor variable for estimating the fuel flow rate. Instead of using the instantaneous aircraft mass as a predictor variable, the mass of the aircraft at takeoff (i.e., the takeoff weight (TOW)) is used as a predictor variable. Aircraft mass is typically not known for a particular flight. The mass of the aircraft changes during flight due to the consumption of fuel. By using the TOW as the predictor variable, the set of unknown predictor variables is restricted to the mass of the aircraft at a single instant (the takeoff instant) rather than the mass of the aircraft at each instant in flight. Moreover, using the TOW as a predictor variable enables simultaneous prediction of the fuel flow rate at all instants in climb out or approach (i.e., batch prediction) leading to faster predictions. Thus, this algorithm of model building is named the 'batch prediction' algorithm. In this paper, it is assumed that the TOW is known (the value recorded in the FDR data is used as an input feature). However, the TOW of a particular flight is not known in practice. In recent work by us, it is demonstrated how GPR can be used to estimate distributions of the TOW from the takeoff roll trajectory variables (*18*).

Using the true values of all input and output variables in the training dataset, GPR models with different kernel functions are trained for each aircraft type, for the climb out and approach phases. All the variables are standardized before training, that is, they are shifted by the sample mean and then scaled by the sample standard deviation of the respective variables in the training datasets. The GPR modeling is done using the MATLAB®-based *GPstuff* toolbox (*19, 20*).

Algorithms using the instantaneous aircraft mass as a predictor variable have also been considered in previous work (*17*). These require a one-step tandem prediction of both the aircraft mass and the fuel flow rate at each time instant. Such algorithms are found to be computationally slower, and yet not give significantly better predictive performance than the batch prediction algorithm, on unseen data. Thus, they are not considered further in this paper and only the batch prediction algorithm is used.

Model Evaluation

The main objective of this paper is the development of a model that predicts the fuel flow rate profile for a flight, given its trajectory data. Given the input variables for a flight, the GPR models produce a predictive distribution of the fuel flow rate. The accuracy of both the point and the interval estimates of the predicted fuel flow rate are evaluated using the following metrics:

- **Mean Absolute Error (MAE)**: This is the mean of the absolute values of the relative prediction errors on the unseen prediction dataset, given by

$$MAE = \frac{1}{n^*}\sum_{i=1}^{n^*}\left|\frac{\hat{\dot{m}}_{f\,i} - \dot{m}_{f\,i,\text{true}}}{\dot{m}_{f\,i,\text{true}}}\right|. \tag{8}$$

Here, n^* is the number of observations in the unseen prediction set, $\dot{m}_{fi,\text{true}}$ is the actual fuel flow rate (ground truth), and $\widehat{\dot{m}}_{fi}$ is the model mean prediction of the fuel flow rate, for the i^{th} observation in the prediction set. The model mean prediction is the mean of the Gaussian fuel flow rate predictive distribution. The MAE therefore reflects the accuracy of the mean prediction (point estimate). A model with a low MAE is desired.

- **Prediction Coverage (PC)**: This is the fraction of observations in the unseen prediction set for which the ground truth values of the fuel flow rates fall within the 95% prediction intervals of the fuel flow rate predictive distributions. The 95% prediction intervals are given by the 95% Highest Density Intervals (HDIs) of the fuel flow rate predictive distributions. The PC indicates how well the prediction intervals capture the variability of the fuel flow rates. A PC value close to 95% increases belief that the model has been properly specified and formulated.
- **Normalized Length of Prediction Interval (NLPI)**: This is the mean of the length of the 95% prediction intervals expressed as a fraction of the corresponding mean fuel flow rate predictions. The NLPI indicates the extent of relative uncertainty present in the predicted fuel flow rate.

For each aircraft type in each phase of flight, GPR models using different kernel functions are trained, and applied to the validation dataset. The evaluation metrics for model predictive performance using different kernels are compared using statistical multiple comparison tests, and the kernel giving the best statistically significant predictive performance (at a 5% significance level) is chosen.

The GPR model with the chosen kernel function is finally evaluated for its predictive performance on the unseen test dataset. The Gaussian predictive distributions given by the GPR models on new inputs in the test data are used to derive the mean predicted fuel flow rates and the 95% prediction intervals for the fuel flow rate. It is worth noting that although GPR models are slow to train, they give fast predictions on new inputs (in the order of a few seconds). In other words, the time taken to predict the fuel flow rate profile for a new flight trajectory is small.

Existing Fuel Burn Models

The statistical models developed are also compared with current state-of-the-practice Aircraft Performance Models (APMs) that are used to estimate fuel burn in systems such as the FAA's Aviation Environmental Design Tool (AEDT). These include:

- **Base of Aircraft Data (BADA) model (*21*)**: Developed by EUROCONTROL, BADA is a total energy-based method used for aircraft performance modeling. It uses simplified equations to model aircraft performance, with different equation coefficients (maintained in the BADA database) for different aircraft types. This method needs aircraft net thrust values to estimate the fuel flow rate. In this paper, net thrust (F_n) values are determined using the BADA Family 3 (revision 3.13) thrust estimation equations (*21*). In this study, BADA 3.13 is used as just a benchmark model. BADA Family 4 models are not explored in this study and could potentially give diferent results than the BADA 3.13 models. The climb out net thrust ($F_{n_{co}}$) is given by the following set of equations:

$$F_{n_{co}} = (1 - C_{Tc,5}\Delta T_{\text{eff}})C_{Tc,1}(1 - \frac{h}{C_{Tc,2}} + C_{Tc,3}h^2)$$
$$\Delta T_{\text{eff}} = \Delta T - C_{Tc,4}$$
$$0 \leq \Delta T_{\text{eff}} \times C_{Tc,5} \leq 0.4$$
$$C_{Tc,5} \geq 0 \qquad (9-12)$$

Here, $C_{T\,c,1}$, $C_{T\,c,2}$, $C_{T\,c,3}$, $C_{T\,c,4}$, and $C_{T\,c,5}$ are thrust coefficients enumerated in the BADA database, and ΔT is the temperature deviation from the ISA. h is the aircraft altitude above mean sea level. The approach net thrust ($F_{n_{ap}}$) is given as:

$$F_{n_{ap}} = \begin{cases} C_{\text{Tdes, high}} \times F_{n_{co}}, & \text{if h>h}_{\text{de}}, \\ C_{\text{Tdes, low}} \times F_{n_{co}}, & \text{if h<h}_{\text{de}} \text{ \& aircraft is in cruise configuration,} \\ C_{\text{Tdes, app}} \times F_{n_{co}}, & \text{if h<h}_{\text{de}} \text{ \& aircraft is in approach configuration,} \\ C_{\text{Tdes, ld}} \times F_{n_{co}}, & \text{if h<h}_{\text{de}} \text{ \& aircraft is in landing configuration.} \end{cases} \qquad (13)$$

Here, $C_{\text{Tdes,high}}$, $C_{\text{Tdes,low}}$, $C_{\text{Tdes,app}}$, and $C_{\text{Tdes,ld}}$ are descent thrust coefficients enumerated in the BADA database. h_{de} is the transition altitude for calculation of descent phase thrust and is enumerated in the BADA database. The definitions of the different configurations are given in the BADA manual. These configurations are treated differently due to different aerodynamic drag values arising from the extension of slats, flaps, and landing gear. Once the net thrust is determined, the following equations are used to estimate the fuel flow rate per engine ($\dot{m}_{f\ \text{BADA}}$):

$$\dot{m}_{f\ \text{BADA}} = \begin{cases} \dfrac{\text{TSFC} \times F_n}{N_{\text{eng}}}, & \text{in climb out, and} \\ \dfrac{1}{N_{\text{eng}}} max(\text{TSFC} \times F_n,\ \dot{m}_{f\,\text{min}}), & \text{in approach.} \end{cases}$$

$$\text{TSFC} = C_{f1}(1 + \frac{V}{C_{f2}})$$
$$\dot{m}_{f\,\text{min}} = C_{f3}(1 - \frac{h}{C_{f4}}) \qquad (14)$$

Here, $\dot{m}_{f\,\text{min}}$ is the minimum aircraft fuel flow rate from all engines. C_{f1}, C_{f2}, C_{f3}, and C_{f4} are aircraft-type specific constants found in the BADA database. In the analysis done here, the values of the input variables to the BADA equations (such as, altitude, temperature, density, true airspeed, etc.) have been obtained from the FDR dataset.

- **Senzig-Fleming-Iovinelli (SFI) model (*4*)**: The SFI model is used in AEDT to model the fuel flow rate in the terminal region of ascent and descent (< 10000' AFE). The fuel flow rate per engine ($\dot{m}_{f\ \text{SFI}}$) is expressed as:

$$\dot{m}_{f\,\text{SFI}} = \begin{cases} (K_1 + K_2 M_\infty + K_3 h + K_4 \frac{F_{n,pe}}{\delta_\infty})\sqrt{\theta_\infty} F_{n,pe}, & \text{in climb out, and} \\ (\alpha + \beta_1 M_\infty + \beta_2 \exp(-\frac{\beta_3 \frac{F_{n,pe}}{\delta_\infty}}{F_{n_0}}))\sqrt{\theta_\infty} F_{n,pe}, & \text{in approach.} \end{cases} \qquad (15)$$

Here, $F_{n,pe}$ is the aircraft net thrust per engine, and F_{n_0} is the engine ISA sea level static thrust. K_1, K_2, K_3, K_4, α, β_1, β_2, and β_3 are aircraft-type specific model coefficients enumerated in the AEDT database. The per-engine net thrust ($F_{n,pe}$) values required as inputs to the SFI model are determined using the BADA thrust estimation equations, Equations 9–13.

- **The ICAO Databank with Boeing Fuel Flow Method 2 Correction (ICAO-BFFM2) model (*22*)**: The ICAO Databank tabulates the values of fuel flow rates in the LTO cycle (*6*). These values are obtained through ground-based uninstalled engine certification tests and the measurements are reported after being corrected to sea level static ISA reference conditions. The databank entry for each engine reports four values for the fuel flow rate, namely, at takeoff (100% thrust, which is seldom true as most takeoffs happen at derated thrust), climb out (85% thrust), approach (30% thrust), and ground idle/taxi (7% thrust, whereas in reality, many idling conditions occur at less than 7% thrust (*23*)). The Boeing Fuel Flow Method 2 (BFFM2) provides corrections to convert the ICAO Databank values to at-altitude conditions for an installed engine (*22*). The fuel flow rate is then given by the following equations:

$$\dot{m}_{f_{\text{ICAO-BFFM2}}} = \begin{cases} 1.013\dot{m}_{f_{\text{ICAO, co}}}\delta_\infty\theta_\infty^{-3.8}\exp(-0.2M_\infty^2), & \text{in climb out, and} \\ 1.020\dot{m}_{f_{\text{ICAO, ap}}}\delta_\infty\theta_\infty^{-3.8}\exp(-0.2M_\infty^2), & \text{in approach.} \end{cases} \qquad (16)$$

Here, $\dot{m}_{f_{\text{ICAO-BFFM2}}}$ is the ICAO-BFFM2 fuel flow rate per engine, and $\dot{m}_{f_{\text{ICAO}}}$ is the reported value of the fuel flow rate for an uninstalled engine at reference conditions in the ICAO Databank. The subscripts "co" and "ap" refer to the reported ICAO Databank values in climb out

Table 1. Fuel Flow Rate: Predictive Performance of the GPR Models and Other Aircraft Performance Models (APMs) in Climb Out and Approach for Different Aircraft Types

			Existing APMs				GPR models	
			Mean absolute error (%)				PC (%)	NLPI (%)
Phase	Aircraft type	Number of flights	BADA	SFI	ICAO	GPR	GPR models	
Climb out	A319-112	130	4.7 (3.3)	8.2 (1.4)	13.0 (5.2)	2.6 (1.0)	95.6 (6.8)	13.9 (1.0)
	A320-214	169	7.3 (4.2)	9.7 (2.8)	8.6 (4.1)	3.5 (2.5)	94.7 (9.8)	20.7 (1.2)
	A321-111	117	11.8 (3.5)	14.6 (2.6)	13.1 (4.1)	4.3 (1.7)	91.7 (9.0)	21.7 (1.2)
	A330-202	84	11.1 (1.6)	10.2 (1.4)	17.3 (2.9)	4.1 (1.8)	91.8 (9.7)	18.3 (1.3)
	A330-243	100	10.3 (1.7)	–	5.9 (3.2)	3.1 (2.2)	91.2 (18.4)	12.9 (0.9)
	A340-541	52	11.7 (6.1)	11.0 (5.6)	13.1 (14.0)	4.7 (3.3)	78.5 (28.5)	14.6 (0.9)
	B767-300	91	5.8 (1.1)	8.9 (1.4)	21.0 (4.4)	2.5 (0.9)	98.4 (3.5)	17.4 (1.1)
	B777-300ER	131	23.0 (5.5)	21.0 (5.8)	23.1 (5.7)	6.7 (1.7)	90.5 (11.6)	32.2 (6.7)
Approach	A319-112	130	24.4 (8.2)	51.0 (9.9)	68.6 (21.9)	17.4 (6.8)	94.7 (4.9)	95.8 (13.5)
	A320-214	169	24.9 (5.9)	75.5 (16.3)	82.2 (20.5)	16.2 (5.1)	94.8 (4.0)	92.8 (14.0)
	A321-111	117	25.7 (7.4)	43.3 (14.8)	58.9 (26.0)	16.4 (4.4)	92.5 (5.4)	77.6 (16.0)
	A330-202	84	46.9 (13.8)	113.0 (22.6)	98.0 (25.8)	30.0 (11.2)	91.2 (7.5)	146.3 (26.5)
	A330-243	100	36.5 (6.5)	–	96.8 (25.8)	20.4 (10.3)	91.5 (8.1)	111.4 (23.1)
	A340-541	52	29.0 (10.2)	42.3 (14.0)	63.1 (20.1)	16.6 (2.6)	95.9 (3.5)	126.0 (24.2)
	B767-300	91	33.6 (6.3)	49.0 (13.2)	78.7 (29.3)	19.3 (7.1)	94.9 (6.1)	126.7 (37.7)
	B777-300ER	131	23.5 (7.3)	39.1 (9.2)	76.2 (23.6)	18.5 (4.7)	94.0 (4.7)	100.6 (16.8)

Note: Each entry shows the mean (and standard deviation within parentheses) of the evaluation metric across all the flights in the test data for that aircraft type. All the evaluation metrics are calculated on destandardized data (that is, data are retained at their original location and scale). The table also shows the total number of flights for each aircraft type in its FDR dataset. 'ICAO' refers to the ICAO-BFFM2 model. There are no entries for the A330-243 SFI models due to the model coefficients being unavailable.

and approach, respectively. The ICAO-BFFM2 method is used in AEDT to model the fuel flow rate in the LTO cycle when engine thrust values or other sources of the fuel flow rate are unavailable.

It is important to note that all the above APMs only provide point estimates of the fuel flow rate, and do not report any uncertainty estimates.

Evaluation of Fuel Flow Rate Models

Table 1 tabulates the predictive performance of the GPR models on the test datasets of the different aircraft types. Each entry shows the mean value (and the standard deviation within parentheses) of the evaluation metric across all the flights in the test dataset. Table 1 also reports the predictive performance of the BADA, SFI, ICAO-BFFM2 models on the test data.

It is observed that for all the aircraft types, the GPR models give a statistically significantly (at 5% significance level) lower mean absolute error than the other APMs. The median of the mean absolute errors across the different aircraft types in climb out is 3.8% for the GPR models, 10.7% for the BADA models, 10.2% for the SFI models, and 13.1% for the ICAO-BFFM2 models. In other words, the GPR models have an at least 62% lower median MAE than the other APMs in climb out. The median of the MAEs across the different aircraft types in approach is 18.0% for the GPR models,

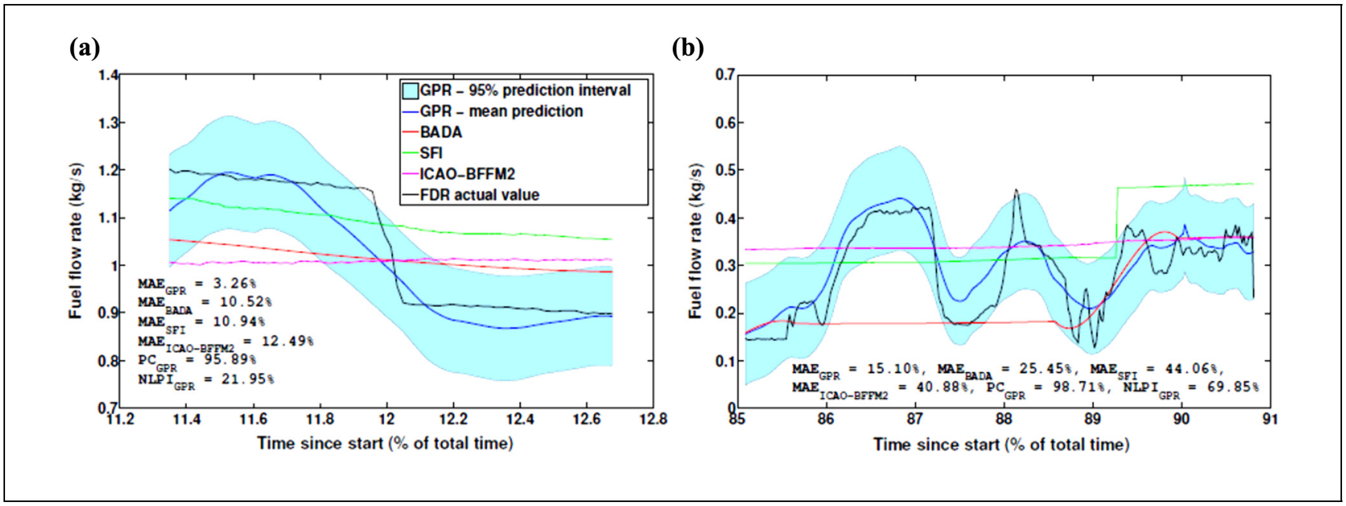

Figure 1. A321-111 fuel flow rate: Predictive performance of the Gaussian Process Regression (GPR), Base of Aircraft Data (BADA), Senzig-Fleming-Iovinelli (SFI), and International Civil Aviation Organization Databank-Boeing Fuel Flow Method 2 (ICAO-BFFM2) models for one test data flight in (*a*) climb out and (*b*) approach. The plots show destandardized fuel flow rates. The plots also show the actual fuel flow rate values from the Flight Data Recorder (FDR) dataset. The scales are different in the two figures. The values of the evaluation metrics (namely, Mean Absolute Error [MAE], Prediction Coverage [PC], and Normalized Length of Prediction Intervals [NLPI]) within the plots are the values for this particular flight alone.

27.4% for the BADA models, 49.0% for the SFI models, and 77.5% for the ICAO-BFFM2 models. The GPR models therefore, have an at least 34% lower median MAE than the other APMs in approach.

Unlike the other APMs which only report mean predictions of the fuel flow rate, the GPR models also give prediction intervals for the fuel flow rate. The median prediction coverage across the different aircraft types is 91.8% in climb out and 94.4% in approach, both of which are close to the nominal coverage probability of 95%. The median NLPI, indicative of the extent of relative variability or uncertainty in the fuel flow rate, across the different aircraft types is 17.9% in climb out and 106.0% in approach. The higher MAE as well as the larger uncertainty estimates in approach are due to the higher operational variability observed in this phase. In contrast to climb out, a typically smooth procedure, the fuel flow rate varies significantly in approach due to operational factors. These observations are illustrated in Figure 1, which shows the predicted and actual fuel flow rates in climb out and approach for one example flight of the A321-111.

Estimation of Fuel Mass Consumed

The fuel flow rate profiles estimated in climb out and approach can be further used to estimate the total mass of the fuel consumed (from all the engines) in climb out and approach. The equation for estimating the total fuel burn in a particular phase of flight is given by

$$\hat{m}_f = N_{\text{eng}} \sum_{i=1}^{n-1} \hat{\dot{m}}_{f\,i} \Delta t_{i+1}. \quad (17)$$

Here, $\hat{m}_f$ is the predicted total mass of fuel consumed in a particular phase of flight, n is the number of time instants in that phase of flight, $\hat{\dot{m}}_{fi}$ is the predicted fuel flow rate per engine for the i^{th} instant, and $\Delta t_{i+1} = t_{i+1} - t_i$ is the time interval between the $(i+1)^{\text{th}}$ and the i^{th} observations. At every instant, the fuel flow rate per engine predicted by a GPR model follows a Gaussian distribution. Therefore, the total mass of fuel consumed (divided by the number of engines and the total time in phase) follows a Gaussian Mixture distribution with mixture component weights governed by the time intervals between successive instants.

Table 2 shows the predictive performance of the total fuel mass estimation models on the entire unseen dataset (not used for training the fuel flow rate models) for the different aircraft types. In climb out, the GPR-based models give a median MAE of 2.0% across the different aircraft types. The median MAEs are 7.6%, 7.5%, and 10.1% for the BADA, SFI, and ICAO-BFFM2 models, respectively. A comparison of the mean absolute errors shows that, for all the aircraft types, the GPR-based models give a statistically significantly lower MAE (at a 5% significance level) than the other APMs. The median MAE in total fuel burn for the GPR-based models is more than 73% lower than the other APMs. The GPR-based models give a median NLPI of 30.6%, with a median PC of 100%. In the approach phase, the GPR-based models give a median MAE of 5.5% across the different aircraft types, compared with values of 13.8%, 42.8%, and 50.6% for the BADA, SFI, and ICAO-BFFM2 models, respectively. The GPR-based models give a

Table 2. Total Mass of Fuel Consumed: Predictive Performance of the GPR-Based Models and Other APMs in Climb Out and Approach for Different Aircraft Types

		Existing APMs			GPR models		
		Mean absolute error (%)				PC (%)	NLPI (%)
Phase	Aircraft type	BADA	SFI	ICAO	GPR	GPR models	
Climb out	A319-112	4.3 (3.5)	6.3 (3.6)	12.7 (4.9)	1.3 (1.3)	100.0 (0.0)	21.8 (3.6)
	A320-214	5.3 (4.4)	6.8 (2.8)	5.9 (4.2)	1.4 (1.1)	100.0 (0.0)	33.7 (5.6)
	A321-111	5.2 (3.2)	7.5 (5.6)	7.2 (5.5)	2.1 (1.5)	97.6 (15.6)	41.1 (8.1)
	A330-202	11.2 (2.0)	5.8 (2.1)	18.1 (3.1)	2.2 (1.7)	93.1 (25.8)	36.8 (4.4)
	A330-243	10.1 (2.1)	–	4.8 (3.3)	1.8 (1.2)	100.0 (0.0)	18.0 (2.9)
	A340-541	9.9 (5.7)	10.0 (5.3)	7.4 (11.8)	2.6 (3.1)	100.0 (0.0)	27.4 (7.1)
	B767-300	5.2 (1.4)	8.3 (1.8)	21.0 (3.6)	1.4 (1.3)	100.0 (0.0)	23.7 (4.6)
	B777-300ER	22.3 (6.5)	13.2 (7.0)	22.1 (7.1)	2.9 (2.1)	73.9 (44.4)	79.4 (9.0)
Approach	A319-112	13.7 (10.8)	42.8 (31.1)	53.3 (36.4)	3.9 (3.6)	100.0 (0.0)	133.6 (16.5)
	A320-214	13.8 (9.7)	63.5 (21.7)	55.9 (20.1)	5.5 (4.4)	100.0 (0.0)	133.6 (18.4)
	A321-111	15.5 (11.5)	31.3 (16.6)	45.8 (23.0)	4.9 (3.4)	100.0 (0.0)	114.3 (21.6)
	A330-202	11.8 (11.2)	82.9 (42.7)	47.8 (42.6)	6.2 (5.7)	100.0 (0.0)	188.9 (22.5)
	A330-243	14.7 (11.9)	–	85.8 (53.8)	7.4 (9.2)	100.0 (0.0)	150.7 (23.5)
	A340-541	24.5 (13.0)	55.2 (89.8)	72.6 (101.0)	5.4 (3.2)	100.0 (0.0)	160.4 (23.3)
	B767-300	13.3 (10.2)	25.7 (21.3)	31.3 (29.0)	4.6 (4.0)	100.0 (0.0)	167.6 (28.4)
	B777-300ER	13.2 (9.9)	22.1 (34.0)	38.1 (37.0)	5.9 (4.2)	100.0 (0.0)	153.2 (22.4)

Note: Each entry shows the mean (and standard deviation within parentheses) of the evaluation metric across all the flights in the test data for that aircraft type. All the evaluation metrics are calculated on destandardized data (that is, data are retained at their original location and scale). The table also shows the total number of flights for each aircraft type in its FDR dataset. 'ICAO' refers to the ICAO-BFFM2 model. There are no entries for the A330-243 SFI models due to the model coefficients being unavailable.

statistically significantly lower or similar MAE (at a 5% significance level) as the other APMs. The GPR-based models achieve a more than 59% reduction in the median MAE in approach. The GPR-based models again give a median PC of 100% in approach, though the extent of uncertainty is higher (as indicated by a median NLPI of 152.0%), which can be attributed to the higher operational variability in this phase.

Discussion and Next Steps

This paper has presented a methodology to statistically estimate aircraft fuel burn in the climb out and approach phases of flight, given trajectory data. Data from flight data recorders of a commercial airline were used for model building, making the models representative of real aircraft operations. The model features important for fuel flow rate estimation were determined using an understanding of the physical principles governing aircraft and engine performance. Modeling assumptions allowed the feature space to be restricted to trajectory variables easily obtainable/derivable from ground-based surveillance data sources.

A nonparametric probabilistic technique called Gaussian Process Regression (GPR) was used to model the fuel flow rate. Under the GPR formulation, the predictive distribution of the fuel flow rate at a new input is a Gaussian distribution, whose parameters (mean and standard deviation) are estimated. The predictive distribution can be used to determine the mean and the 95% prediction intervals for the fuel flow rate. The mean gives a point estimate of the fuel flow rate, while the prediction intervals quantify its uncertainty resulting from random disturbances, model assumptions, and unmodeled features, and the operational variability seen in fuel flow rates. The estimated fuel flow rates can be further used to determine the predictive distributions of the total fuel consumed in climb out or approach. The statistical models were compared with other commonly used APMs. It was found that the GPR-based models gave a better predictive performance for the fuel flow rates and the mass of fuel consumed.

The statistical models developed can be used to estimate the fuel burn and emissions impact of real flight trajectories, and default operational procedures. As a result, the user of these models can more realistically model fuel burn and emissions in the vicinity of airports. These models will help improve fuel burn and emission inventories by evaluating impacts on a per-flight operation basis (instead of on a fleet/airline/regional level alone). Our models estimate the fuel burn using trajectory information, thereby removing the need to estimate any hidden variables. This presents a significant advantage over other APMs, which often require an estimate of the net thrust, a challenging problem in itself. In contrast to other fuel burn models, statistical models provide a predictive distribution, rather than a point estimate. They therefore quantify the expected uncertainty in fuel burn; these uncertainty estimates can be used to identify anomalous behavior. More detailed discussions of these results can be found in (*24*).

The proposed models suffer from the common disadvantage presented by any data-driven model: They assume that the training dataset is representative of the range of operating points of interest. Therefore, they cannot be applied to input variable values that are radically different from those seen in the training data. It should be noted that the models developed in this paper were trained on a relatively small subset of flight data. Models trained on larger sets of data, covering a variety of operating environments, airports, airlines and aircraft types, would be more generalizable. Given such new training data, the methodology outlined in this paper can easily be used to develop new models. Data used for model building in this paper were not differentiated on the basis of airports. The impact of an airport on fuel burn can be assessed by developing airport-specific models. Similarly, additional features such as flight distances, and high-fidelity weather data may also be investigated. Finally, more precise estimates of the fuel mass consumed can be obtained by modeling it directly, rather than by aggregating fuel flow rate estimates.

Acknowledgments

This research was supported by the TRB through the 2016–2017 Graduate Research Award Program on Public-Sector Aviation Issues. The authors thank Monica Alcabin, Bob Arnott, Brian Kim, Larry Goldstein, and Saadat Syed for their insights and feedback, as well as Mary Sandy and Sarah Pauls of the Virginia Space Grant Consortium for logistical support. This research was also supported in part by the National Science Foundation under grant number 1239054, CPS:Frontiers:FORCES.

References

1. FAA Office of Environment and Energy. *Aviation Emissions, Impact, and Mitigation: A Primer*. FAA, Washington, D.C., 2015.
2. Owen, B., D. S. Lee, and L. Lim. Flying into the Future: Aviation Emissions Scenarios to 2050. *Environmental Science and Technology*, Vol. 44, No. 7, 2010, pp. 2255–2260.
3. GasTurb 12 software [Online], 2015. http://gasturb.de/. Accessed October 2015.
4. Senzig, D. A., G. G. Fleming, and R. J. Iovinelli, Modeling of Terminal-Area Airplane Fuel Consumption. *AIAA Journal of Aircraft*, Vol. 46, No. 4, 2009, pp. 1089–1093.
5. Trani, A. A., F. C. Wing-Ho, G. Schilling, H. Baik, and A. Seshadri. A Neural Network Model to Estimate Aircraft Fuel Consumption. *Proc., AIAA 4th Aviation Technology, Integration and Operations (ATIO) Forum*, Chicago, IL, 2004.
6. International Civil Aviation Organization. *ICAO Aircraft Engine Emissions Databank* [Online], 2014. https://easa.europa.eu/document-library/icao-aircraft-engine-emissions-databank. Accessed February 2014.
7. Chati, Y. S., and H. Balakrishnan. Analysis of Aircraft Fuel Burn and Emissions in the Landing and Take off Cycle using Operational Data. *Proc., 6th International Conference on Research in Air Transportation*, Istanbul, Turkey, 2014.
8. Enea, G., J. Bronsvoort, H. Fricke, C. Seiß, J. Rosenow, A. Ramadani, and M. Paglione. Fuel Burn Estimation Modeling for ATM Benchmark Applications Perspectives from an International Collaboration. In *USA/Europe Air Traffic Management Research and Development Seminar*, Seattle, WA, 2017.
9. Kulikov, G. G., and H. A. Thompson. (eds.). *Dynamic Modelling of Gas Turbines: Identification, Simulation, Condition Monitoring, and Optimal Control*. Springer-Verlag, London, 2004.
10. Chu, E., D. Gorinevsky, and S. P. Boyd. Detecting Aircraft Performance Anomalies from Cruise Flight Data. *Proc., AIAA Infotech @ Aerospace Conference*, Atlanta, GA, 2010.
11. Hastie, T., R. Tibshirani, and J. Friedman. *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*. Springer-Verlag, New York, 2009.
12. Rasmussen, C. E., and C. K. I. Williams. *Gaussian Processes for Machine Learning*. The MIT Press, Cambridge, Mass., 2006.

13. Yoder, T. *Development of Aircraft Fuel Burn Modeling Techniques with Applications to Global Emissions Modeling and Assessment of the Benefits of Reduced Vertical Separation Minimums*. Master's thesis. Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge, 2007.
14. Cumpsty, N. A. *Jet Propulsion: A Simple Guide to the Aerodynamics and Thermodynamic Design and Performance of Jet Engines* (2nd ed.). Cambridge University Press, Cambridge, UK, 2003.
15. Dürichen, R., M. A. Pimentel, L. Clifton, A. Schweikard, and D. A. Clifton. Multi-Task Gaussian Process Models for Biomedical Applications. *Proc., IEEE-EMBS International Conference on Biomedical and Health Informatics*, Valencia, Spain, IEEE, New York, 2014.
16. Platt, J. C., C. J. C. Burges, S. Swenson, C. Weare, and A. Zheng. Learning a Gaussian Process Prior for Automatically Generating Music Playlists. In *Advances in Neural Information Processing Systems 14* (T. G. Dietterich, S. Becker, and Z. Ghahramani, eds.), MIT Press, Cambridge, MA, 2002, pp. 1425–1432.
17. Chati, Y. S., and H. Balakrishnan. A Gaussian Process Regression Approach to Model Aircraft Engine Fuel Flow Rate. *Proc., ACM/IEEE International Conference on Cyber-Physical Systems*, Pittsburgh, Pa., IEEE, New York, 2017.
18. Chati, Y. S., and H. Balakrishnan. Statistical Modeling of Aircraft Takeoff Weight. In *USA/Europe Air Traffic Management Research and Development Seminar*, Seattle, WA, 2017.
19. The Mathworks, Inc. *MATLAB® R2012a Documentation*. Software Package, 2012.
20. Vanhatalo, J., J. Riihimäki, J. Hartikainen, P. Jylänki, V. Tolvanen, and A. Vehtari. GPstuff: Bayesian Modeling with Gaussian Processes. *Journal of Machine Learning Research*, Vol. 14, 2013, pp. 1175–1179.
21. Nuic, A. *User Manual for the Base of Aircraft Data (BADA) Revision 3.13*. EEC Technical/Scientific Report No. 15/04/02-43. EUROCONTROL Experimental Centre, 2015.
22. Ahearn, M., E. Boeker, S. Gorshkov, A. Hansen, S. Hwang, J. Koopmann, A. Malwitz, G. Noel, C. Reherman, D. Senzig, G. B. Solman, Y. Tosa, A. Wilson, A. Zubrow, N. Didyk, J. DiPardo, F. Grandi, M. Majeed, J. Bernal, E. Dinges, D. Rickel, M. Yaworski, C. Hall, and S. Augustineetal />. *Aviation Environmental Design Tool (AEDT) Technical Manual, Version 2b, Service Pack 3*. Report No. DOT-VNTSC-FAA-16-11. U.S. Department of Transportation Volpe National Transportation Systems Center, 2016.
23. Herndon, S. C., T. Rogers, E. J. Dunlea, J. T. Jayne, R. Miake-Lye, and B. Knighton. Hydrocarbon Emissions from In-Use Commercial Aircraft during Airport Operations. *Environmental Science and Technology*, Vol. 40, No. 14, 2006, pp. 4406–4413.
24. Chati, Y. S. *Statistical Modeling of Aircraft Engine Fuel Burn*. Ph.D. thesis. Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge, MA, 2018.

The ACRP Selection Panel for the Graduate Research Award Program on Public-Sector Aviation Issues peer-reviewed this paper (18-01539).