On the Probabilistic Modeling of Runway Inter-departure Times

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Abstract. This paper examines the validity of the Erlang distribution for runway service times. It uses high-fidelity surface surveillance data, for the first time, to model the probability distributions of runway service times and departure throughput, and to validate the Erlang service time assumption. The paper proposes several potential approaches to determine departure runway service time distributions from empirical data, and compares the results. In particular, it finds that a displaced exponential fit may be a better match to the empirical service time distribution than an Erlang distribution. However, the computational benefits offered by the Erlang service time distribution, its accurate reflection of the means and variances of the empirical service time and throughput distributions, and its ability to represent the tail of the service time distribution, make it attractive for use in queuing models of airport operations.

Keywords. runways, inter-departure times, modeling, probability distribution

Introduction

The Erlang distribution has been widely used to model runway service process in queuing models of air traffic operations [1,2,3], since its use was first proposed by Hengsbach and Odoni [4]. The Erlang distribution has been shown to offer certain computational advantages, because it can be viewed as a sum of exponential distributions. To the best of our knowledge, there have been few efforts to validate Erlang distribution assumptions using operational data, and even those have only been performed informally with aggregate or low-fidelity data [5]. This paper attempts to use high-fidelity surface surveillance data to examine the validity of the Erlang distribution for runway service times.

1. Data sources

The Aviation System Performance Metrics (ASPM) database maintained by the Federal Aviation Administration (FAA) provides records of the wheels-off and wheels-on times of all domestic flights in the United States (US) [6]. These reports are obtained automati-

cally through a system called ACARS for the major carriers, and are inferred for the others [7]. While a valuable data source, ASPM presents several challenges for validating actual runway service time distributions:

- Aircraft takeoff times are rounded to the nearest minute [7]. Due to this quantization, aircraft can appear to be taking off with zero inter-departure time from the same runway.
- To estimate service times from departure times, periods of persistent runway demand must be extracted from the ASPM data [8]. In particular, it is difficult to use this low-fidelity data to identify instances when there was at least one aircraft present in the queue at the departure runway threshold.

The Airport Surface Detection Equipment – Model X (ASDE-X) system combines data from surface radar tracks, multilateration and ADS-B where available to present a more detailed view of airport surface operations [9]. Aircraft wheels-off times are typically captured with a precision of seconds. In addition, ASDE-X data can be used to measure the precise number of aircraft that are physically present in the queuing area at the departure runway threshold [10,11]. As a result, runway inter-departure times can be measured conditioned on the actual state of the departure queue.

For the above reasons, ASDE-X data presents a promising way to accurately model runway service times, and to assess the validity of the Erlang distribution to represent them. This paper presents a case study based on operations at Boston Logan International Airport (BOS), for the 22L, 27 | 22L, 22R runway configuration in the year 2011.

2. Fitting of Erlang distributions

The first step in the analysis is the extraction of instances in which there was persistent demand, and to fit an Erlang distribution to the inter-departure times seen during these instances. Persistent demand in this case is identified by 15 min periods when there were more than 22 aircraft taxiing out [8]. The resultant empirical throughput distribution over these instances is shown in red in Figure 1.
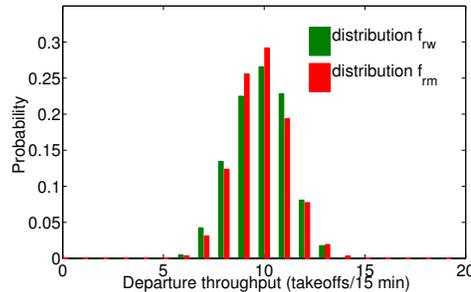

Figure 1. Empirical (f_{rw}) and modeled (f_{rm}) probability distributions of the departure throughput of 22L, 27 | 22L, 22R at BOS.

Suppose the service times were generated from an Erlang distribution with parameters $(k, k\mu)$, where $k \in \mathbb{N}_+$ and $k\mu > 0$ are the shape and rate parameters, respectively. In

other words, the probability density function of the service times is assumed to be given by

$$f(t;\, k, k\mu) = \frac{(k\mu)^k t^{k-1} e^{-k\mu t}}{(k-1)!},\ t > 0 \tag{1}$$

The parameters of the Erlang distribution are estimated from the data using an approximation based on the method of moments. Let μ_1 and μ_2 denote the first and second moment of the empirical runway throughput distribution f_{rw}. Since the runway service times are drawn from the Erlang distribution $(k, k\mu)$, if there are exactly i takeoffs in the time interval Δ, there are at least $(i-1)k+1$ and no more that $(i+1)k-1$ occurrences of a Poisson random variable with rate $k\mu\Delta$. In the first case, the first $k-1$ occurrences corresponding to the ith takeoff occurred in the previous time period, in the latter, the last $k-1$ occurrences correspond to the $(i+1)$th takeoff which takes place in the next time period. Summing over all these possibilities yields the following expressions:

$$\mu_1 = \sum_{i=0}^{\infty} \left(i \cdot \sum_{j=(i-1)k+1}^{(i+1)k-1} \frac{k - |ik - j|}{k} \cdot e^{(-k\mu \cdot \Delta)} \cdot \frac{(k\mu \cdot \Delta)^j}{j!} \right) \tag{2}$$

$$\mu_2 = \sum_{i=0}^{\infty} \left(i^2 \cdot \sum_{j=(i-1)k+1}^{(i+1)k-1} \frac{k - |ik - j|}{k} \cdot e^{(-k\mu \cdot \Delta)} \cdot \frac{(k\mu \cdot \Delta)^j}{j!} \right) \tag{3}$$

The method of moments cannot be applied exactly because k is constrained to be a natural number. For this reason, the following approximation is made: The parameter μ is obtained by numerically solving Equation (2) as a function of increasing values of k. For each pair $(\mu, k\mu)$, the error of Equation (3) is calculated. The iterations are stopped when the absolute error increases. Any further increase in k would imply a further decrease in variance and a larger absolute error in the value of the second moment.

The actual, f_{rw}, and fitted, f_{rm}, distribution parameters are shown in Table 1. The parameters $(k, k\mu)$ of the Erlang distribution, g_{rm}, yielding the throughput distribution f_{rm} are (6, 3.92). The Erlang distribution g_{rm} has an average service time of 1.5 min with variance 0.4 min^2.

Table 1. Empirical and modeled runway throughput distributions, f_{rw} and f_{rm}, for 22L, 27 | 22L, 22R at BOS.

Distribution	f_{rw}	f_{rm}
Mean (aircraft/15 min)	9.81	9.81
Standard deviation (aircraft/15 min)	1.38	1.34

3. Estimation of empirical service times

Consistent with Air Traffic Control phraseology, a queue *with pressure* is defined as a departure queue with a sufficient number of aircraft such that that aircraft take off as soon

as the runway is available. Conditioned on a queue with pressure, the service time equals the inter-departure time. For estimating the condition that implies a queue *with pressure*, we use the following algorithm, where $d_q(l)$ is defined as the departure queue at the time of takeoff of the lth aircraft, and $Z = \max d_q(l)$:

for $i = 1 \rightarrow Z$ **do**
    $S_i \leftarrow$ set of inter-departure times of aircraft for which $d_q = i$
end for
$i \leftarrow 1$
$C \leftarrow 0$
while $C \neq 1$ **do**
    if inter-departure time distributions of sets $S_i, \ldots, S_Z$ are statistically significantly different **then**
        $i \leftarrow i+1$
    else
        $C \leftarrow 1$
    end if
end while
return i

In other words, the algorithm identifies the value of d_q for which the inter-departure times do not change significantly with the number of aircraft in the queue. A non-parametric method, namely, the Kruskal-Wallis one-way analysis of variance, is used to compare the distributions of service time distributions in sets $S_i, \ldots, S_Z$. In this particular case, the algorithm returns a value of 5, implying that the inter-departure times for this configuration at BOS are distributed differently when there are 4 aircraft in queue and when there are 5 or more aircraft in queue. Such a large number of aircraft in the queue area is necessary to guarantee that the trailing aircraft is at the runway threshold, and not traveling through the queuing area or on hold for extraneous reasons.

For the aircraft in sets $S_5, S_6 \ldots S_Z$, the inter-departure time equals the service time. It is worth noting that the obtained set of service times does not consist of independent samples, and two subsequent takeoffs may be correlated. As shown in earlier work, one Heavy aircraft departure within a 15-minute interval does not impact the departure throughput significantly, because the controllers use the wake vortex separation to perform runway crossings [12]. As a result, the inter-departure times of surrounding non-Heavy aircraft may not be significantly impacted. In such a scenario, the service times are correlated. Similarly, when there no Heavy aircraft in the departure queue, a controller might choose to perform a stream of tight non-Heavy departures followed by a stream of runway crossings. In this case, the service times of the departures will be correlated. This issue has been previously recognized, and capacity is typically defined over a long time period (for instance the *saturation capacity*, the *practical hourly capacity* and the *sustained capacity* are all defined over an hourly-time window) [13]. The 15-minute period has been found to provide a good compromise between sufficient length and persistent demand through the duration of the time period [12,14,15,16].

In order to get independent samples of service time distributions, sets of service times that are 15 min apart are randomly sampled. The 15 min value was chosen for consistency with the throughput estimates, which are also performed for 15 min time windows.
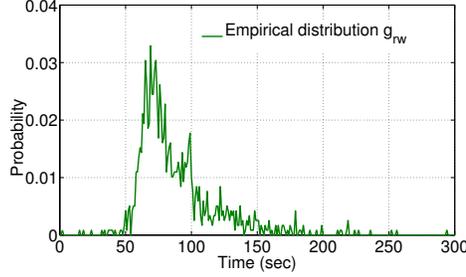
Figure 2. Empirical service time probability distribution for departures of 22L, 27 | 22L, 22R at BOS.

The empirical distribution for the service times of departures l with $d_q(l) = 5$, and which takeoff at least 15 min apart, is shown in Figure 2. The figure shows that the service time distributions have a very long tail despite having a queue with pressure. The support of the distribution is seen to start around 50 sec, and not 60 sec as would be expected. The reason for this difference is that inter-departure times are measured at wheels-off, and not at the start of the takeoff roll where separation is applied by the controllers. The mode of the distribution is found to be 68 sec. The distribution exhibits also a second distinct peak at around 100 sec, which can be attributed to Heavy aircraft departures.

4. Probabilistic modeling of service times

In this section, four different fits to the empirical service time distributions are compared and evaluated:

1. The maximum likelihood estimate (MLE) Gamma distribution (g_{gl}), which estimates the the maximum likelihood parameters of a Gamma distribution to fit the empirical distribution in Figure 2.
2. The displaced exponential distribution fit (g_{de}), given by

$$g_{de}(x; \phi, d) = \begin{cases} \phi \cdot e^{-\phi(x-d)} & \text{if } x \geq d \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$

The displaced exponential distribution is often used in traffic engineering applications because it assumes that there is a minimum headway (d) between vehicles, in addition to a probabilistic quantity [17]. It could potentially be a good model for the runway service time distribution, since it reflects the minimum required separation between successive departures.

In fitting the displaced exponential distribution, the parameters (ϕ, d) are chosen using the Method of Moments:

$$d + \frac{1}{\phi} = \mathbb{E}[S] \qquad (5)$$

$$\frac{1}{\phi^2} = \text{var}(S) \qquad (6)$$

3. The Erlang distribution fit from applying an approximate method of moments (MoM), g_{em}. In this method, the Method of Moments is first used to fit a gamma distribution to the observed service times. The Method of Moments for the gamma distribution yields estimates $\hat{k}$ and $\hat{\lambda}$ for the shape and scale parameters as follows:

$$\hat{k} = \frac{(\mathbb{E}[S])^2}{\text{var}(S)} \tag{7}$$

$$\hat{\lambda} = \frac{\mathbb{E}[S]}{\text{var}(S)} \tag{8}$$

The shape parameter, k, is then constrained to be an integer, in order to transform the Gamma distribution to an Erlang one. The objective is to find an Erlang distribution which has the same mean as the observed distribution, and shape (k) that will result in a variance as close as possible to the observed one.

$$\hat{k} = \lfloor \frac{(\mathbb{E}[S])^2}{\text{var}(S)} + 0.5 \rfloor \tag{9}$$

$$\hat{\lambda} = \frac{\lfloor \frac{(\mathbb{E}[S])^2}{\text{var}(S)} + 0.5 \rfloor}{\mathbb{E}[S]} \tag{10}$$

The resulting Erlang distribution has a mean $\mathbb{E}[L_k]$ and variance $\sigma^2_{L_k}$ such that

$$\mathbb{E}[L_k] = \frac{\hat{k}}{\hat{\lambda}} = \mathbb{E}[S] \tag{11}$$

$$\sigma^2_{L_k} = \frac{\hat{k}}{\hat{\lambda}^2} = \frac{\mathbb{E}[S]^2}{\lfloor \frac{(\mathbb{E}[S])^2}{\text{var}(S)} + 0.5 \rfloor} \approx \text{var}(S) \tag{12}$$

4. The Erlang distribution fit g_{rw} which was obtained by fitting f_{rm} to f_{rw}, as seen in Figure 1. The parameters in the current case were estimated to be (6, 3.92). While f_{rw} comprises all departure throughput observations in saturation (more than 22 departures taxiing out), the service time distribution shown in Figure 2 comprises independent samples of inter-departure times given a queue with pressure. The Erlang distributions g_{rw} and g_{em} essentially model the same quantity, but are estimated differently. Some differences are expected between the two distributions, because the empirical distribution g_{rw} is sampled randomly; a different sampling could yield different parameters.

5. Results

Figure 3 shows the results of applying the four different fitting procedures to ASDE-X data from BOS, for the 22L, 27 | 22L, 22R configuration in 2011. The estimated parameters for the four distributions are shown in Table 2.

The plots suggest that the displaced exponential fit matches the empirical distribution best. d is estimated to be 0.88 min (53 sec), suggesting that it accurately captures the minimum separation requirement. The displaced exponential fit also matches the tail
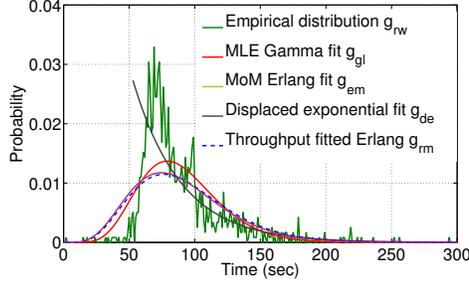
Figure 3. Service time probability distribution fits for departures of runway configuration 22L, 27 | 22L, 22R at BOS.

Table 2. Distribution parameters

Distribution	Parameter 1 (Shape/ Displacement)	Parameter 2 (Rate)	Mean	Var.
Empirical	–	–	1.49	0.30
g_{gl}	8.54	5.72	1.49	0.26
g_{de}	0.88	0.62	1.49	0.30
g_{em}	6	4.02	1.49	0.37
g_{rw}	6	3.92	1.53	0.39

of the empirical distribution very well. However, it does not predict the mode of the distribution exactly.

The Gamma and Erlang fits also fail to predict the mode of the distribution, and they overestimate the density of the distribution for values lower than 60 sec. By contrast, they predict the tails of the empirical distribution to the same extent as the displaced exponential fit. The Gamma and Erlang distributions are different, as can be verified from their parameters in Table 2. The discrepancy is not the result of rounding $\hat{k}$ (it is rounded up to 6 from 5.98), but due to the different estimation methods applied (MLE versus MoM).

Finally, distributions g_{em} and g_{rw} are seen to be very similar despite being derived differently. The similarity between the two distributions demonstrates that the estimated departure throughput under persistent demand and the inter-departure times given a queue with pressure are consistent with each other.

Estimating the service time distribution from the throughput distribution, as outlined in Section 2, appears to accurately calculate not only the mean and the variance of the departure throughput, but also the mean and variance of the inter-departure time given a queue with pressure. On the other hand, Figure 3 suggests that the displaced exponential is a better fit to the empirical service times than an Erlang distribution. This hypothesis is tested by using the estimated parameters $(k, k\mu)$ of the fitted f_{rm} distribution to derive a displaced exponential distribution with the same mean and variance as follows:

Table 3. Comparison of the distributions f_{rw}, f_{rm} and f_{de}.

Distribution	Mean	Variance	KL Divergence
f_{rw}	9.81	1.90	–
f_{rm}	9.81	1.80	0.010
$\tilde{f}_{de}$	9.81	1.77	0.008

$$\tilde{g}_{de}(x; \tilde{\phi}, \tilde{d}) = \begin{cases} \tilde{\phi} \cdot e^{-\tilde{\phi}(x-\tilde{d})} & \text{if } x \geq \tilde{d} \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

such that:

$$\tilde{d} + \frac{1}{\tilde{\phi}} = \frac{1}{\mu} \tag{14}$$

$$\frac{1}{\tilde{\phi}^2} = \frac{1}{k\mu^2} \tag{15}$$

For this example, the parameters are calculated to be (0.91, 0.62), which are, as expected, similar to those of g_{de} (Table 2). The corresponding $\tilde{f}_{de}$ is shown in Figure 4 and is a good match to the empirical distribution. Table 3 shows that $\tilde{f}_{de}$ has a smaller Kullback Leibler (KL) divergence (a measure of the difference between two probability distributions [18]) from the actual throughput distribution, when compared to f_{rm}. It is therefore conjectured that $\tilde{g}_{de}$ accurately represents the service time distribution. It models the minimum service time requirement, the observed tail of the empirical service time distribution, as well as the associated departure throughput distribution.
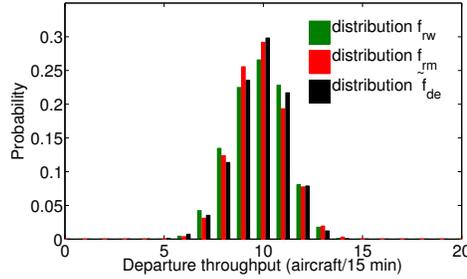

Figure 4. Empirical departure throughput distribution, f_{rw}, and fits f_{rm} and $\tilde{f}_{de}$ for 22L, 27 | 22L, 22R at BOS.

6. The need for sampling

This section illustrates the need to sample the runway service times. Figure 2 shows the empirical distribution g_{rw} of the sampled service times, given a queue with pressure. Suppose this distribution is used to generate the corresponding throughput distribution, f_{sf} in a 15-minute period. Consider the alternative distribution g_{sa} of all service times, given a queue with pressure. The corresponding throughput distribution, f_{sa}, in a 15-minute period can be similarly generated.
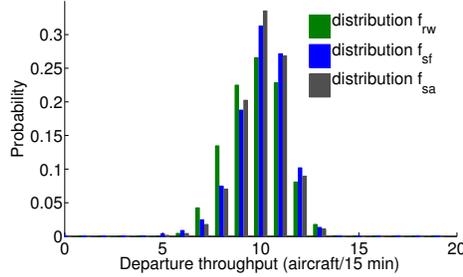
Figure 5. Empirical throughput distribution, f_{rw}, and fits f_{sf} and f_{sa} for departures of BOS runway configuration 22L, 27 | 22L, 22R.

Table 4. Comparison of the distributions f_{rw}, f_{sf} and f_{sa}.

Distribution	Mean	Variance	KL Divergence
f_{rw}	9.81	1.90	–
f_{sf}	10.05	1.78	0.0445
f_{sa}	10.06	1.49	0.1053

Figure 5 compares f_{sf} and f_{sa} to the empirical distribution f_{rw}. The three distributions are also compared in Table 4. The comparisons show that f_{sa} is a less variable throughput distribution than f_{rw}, due to its using dependent observations. Table 4 shows that f_{sa} has a higher KL-distance from f_{rw} than f_{sf} has. The above observations illustrate the need for sampling the inter-departure times in order to get independent observations.

7. Effect of fleet mix on service times

The empirical service time distributions can be further parametrized by fleet mix. Heavy jets are expected to have a service time of at least 2 minutes (120 sec). Figure 6 shows the empirical service time distributions parametrized by the type of aircraft taking off. The distribution of the service times for Heavy jet aircraft is clearly distinct from that of the non-Heavy jets. Heavy jets have much longer service times, as expected given their separation requirement. The mode of the distribution is at 105 sec. Since the service time is measured as the difference between successive wheels-off times, it is less on average than the required separation at the start of the takeoff roll, since Heavy aircraft tend to have longer roll times. The average service time of non-Heavy aircraft is 87 sec, compared to 119 sec for a Heavy aircraft.

Finally, these findings are compared to estimates of jet departure capacity presented in earlier work [19]. In that work, it was concluded that the departure throughput does not depend on Heavy aircraft, whereas Figure 6 shows that Heavy aircraft tend to be separated from the subsequent departures for much longer than non-Heavy aircraft. However, Figure 6 does not show the impact of Heavy departures on the service times of surrounding non-Heavy aircraft. In Figure 6, this would imply that short non-Heavy aircraft service times are correlated with Heavy departures in the surrounding time-window. Similarly, the impact of an arrival bank will not be seen in the inter-departure time of a single aircraft, but will be seen in the 15-min departure throughput.
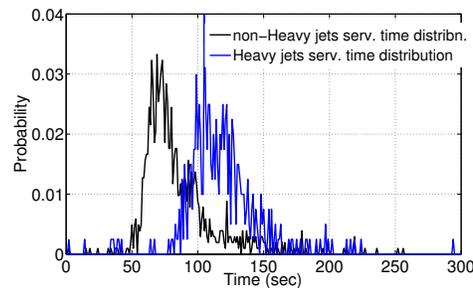
Figure 6. Empirical service time probability distributions for departures of BOS runway configuration 22L, 27 | 22L, 22R for Heavy and non-Heavy jets.

8. Conclusions

This paper presents the first attempt to evaluate the validity of the Erlang distribution for runway service times, using surface surveillance data. The analysis shows that while the Erlang distribution may provide computational advantages, the displace exponential distribution may be a better match to the empirical service time distribution. The complete modeling of separation requirements and the impact of the exogenous variables (arrival crossings, route availability, fanning of props, etc.) would need a hidden Markov model in which the exogenous variables along with the endogenous one (Heavy vs. non-Heavy) would explain the separation times. The complexity of such model renders it impractical for both modeling and simulation. The analysis proposed in this paper has implications both to the development of queuing models of airport operations, and to the estimation of airport capacities.

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